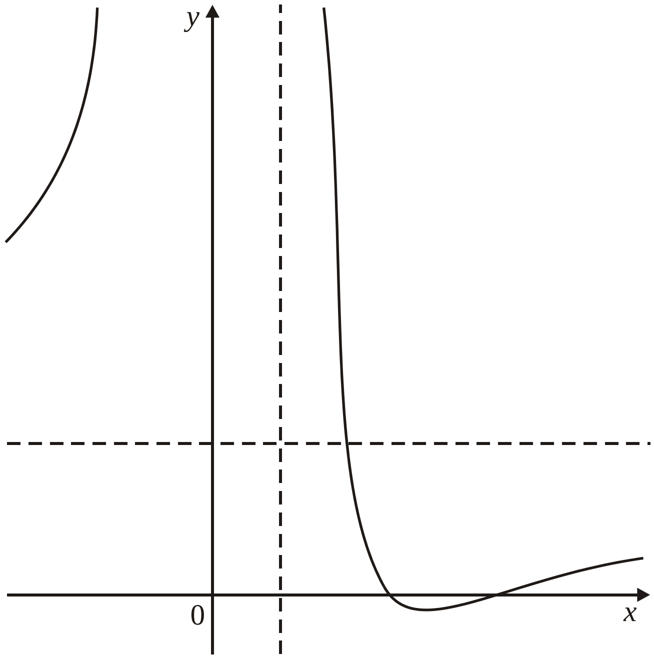
Name:__________________

Answer the following questions :-

1. Consider the function f given by $f(x) = \dfrac{2x^2 - 13x + 20}{(x-1)^2}$, $x \neq 1$.

A part of the graph of f is given below.



The graph has a vertical asymptote and a horizontal asymptote, as shown.

(a) Write down the **equation** of the vertical asymptote.

(b) $f(100) = 1.91$ $f(-100) = 2.09$ $f(1000) = 1.99$

(i) Evaluate $f(-1000)$.

(ii) Write down the **equation** of the horizontal asymptote.

(c) Show that $f'(x) = \dfrac{9x-27}{(x-1)^3}$, $x \neq 1$.

The second derivative is given by $f''(x) = \dfrac{72-18x}{(x-1)^4}$, $x \neq 1$.

(d) Using values of $f'(x)$ and $f''(x)$ explain why a minimum must occur at $x = 3$.

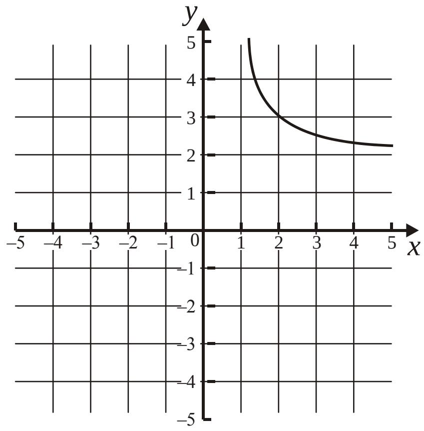
(e) There is a point of inflexion on the graph of f. Write down the coordinates of this point.

2. Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$.

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$.

3. (a) Consider the function $f(x) = 2 + \frac{1}{x-1}$. The diagram below is a sketch of part of the graph of $y = f(x)$.



Copy and complete the sketch of $f(x)$.

(b) (i) Write down the x-intercepts and y-intercepts of $f(x)$.

(ii) Write down the equations of the asymptotes of $f(x)$.

(c) (i) Find $f'(x)$.

(ii) There are no maximum or minimum points on the graph of $f(x)$.
Use your expression for $f'(x)$ to explain why.

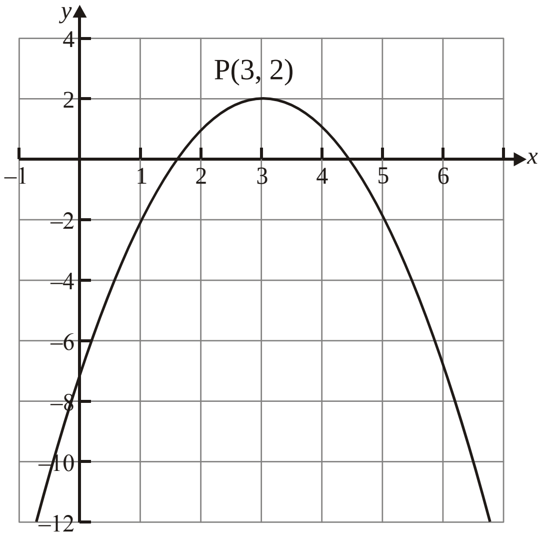
4. The function $f(x)$ is defined as $f(x) = -(x - h)^2 + k$. The diagram below shows part of the graph of $f(x)$. The maximum point on the curve is P $(3, 2)$.

(a) Write down the value of

(i) h; (ii) k.

(b) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$.

(c) Find $f'(x)$.



The point Q lies on the curve and has coordinates (4, 1). A straight line L, through Q, is perpendicular to the tangent at Q.

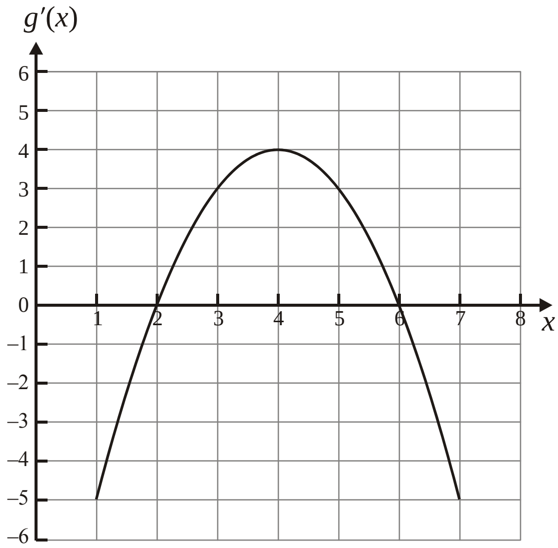
(d) (i) Calculate the gradient of L.

(ii) Find the equation of L.

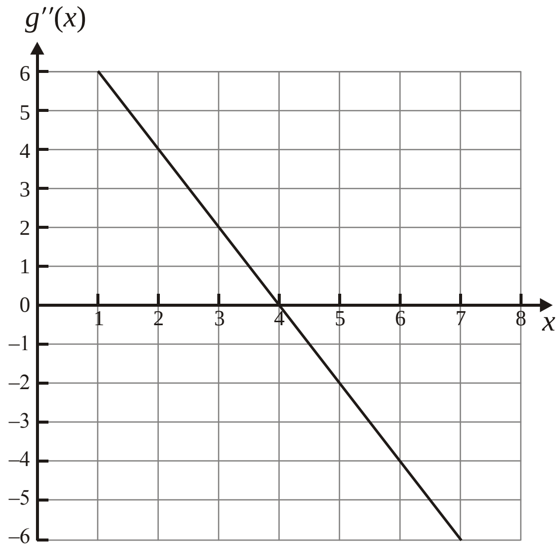
(iii) The line L intersects the curve again at R. Find the x-coordinate of R.

5. Let $y = g(x)$ be a function of x for $1 \le x \le 7$. The graph of g has an inflexion point at P, and a minimum point at M.

Partial sketches of the curves of g' and g'' are shown below.



$y = g'(x)$



$y = g''(x)$

Use the above information to answer the following.

(a) Write down the x-coordinate of P, **and** justify your answer.

(b) Write down the x-coordinate of M, **and** justify your answer.

(c) Given that $g(4) = 0$, sketch the graph of g. On the sketch, mark the points P and M.

6. The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.

(a) Write down

(i) $f'(x)$; (ii) $f'(0)$.

(b) Let N be the normal to the curve at the point where the graph intercepts the y-axis. Show that the equation of N may be written as $y = -0.5x + 2.5$.

Let $g : x \mapsto -0.5x + 2.5$

(c) (i) Find the solutions of $f(x) = g(x)$.

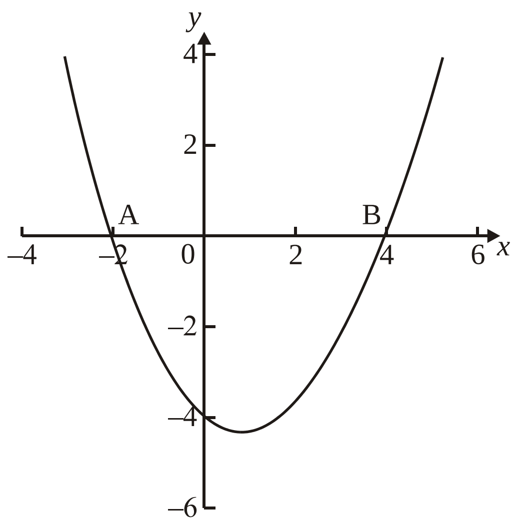
(ii) Hence find the coordinates of the other point of intersection of the normal and the curve.

(d) Let R be the region enclosed between the curve and N.

(i) Write down an expression for the area of R.

(ii) Hence write down the area of R.

7. The equation of a curve may be written in the form $y = a(x - p)(x - q)$. The curve intersects the x-axis at A(–2, 0) and B(4, 0). The curve of $y = f(x)$ is shown in the diagram below.



(a) (i) Write down the value of p and of q.

(ii) Given that the point (6, 8) is on the curve, find the value of a.

(iii) Write the equation of the curve in the form $y = ax^2 + bx + c$.

(b) (i) Find $\frac{dy}{dx}$.

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 7. Find the coordinates of P.

(c) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point B.

(i) Find the equation of L.

(ii) Find the x-coordinate of the point where L intersects the curve again.

8. The function $g(x)$ is defined for $-3 \le x \le 3$. The behaviour of $g'(x)$ and $g''(x)$ is given in the tables below.

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	negative	0	positive	0	negative

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	positive	0	negative

Use the information above to answer the following. In each case, justify your answer.

(a) Write down the value of x for which g has a maximum.

(b) On which intervals is the value of g decreasing?

(c) Write down the value of x for which the graph of g has a point of inflexion.

(d) Given that $g(-3) = 1$, sketch the graph of g. On the sketch, clearly indicate the position of the maximum point, the minimum point, and the point of inflexion.