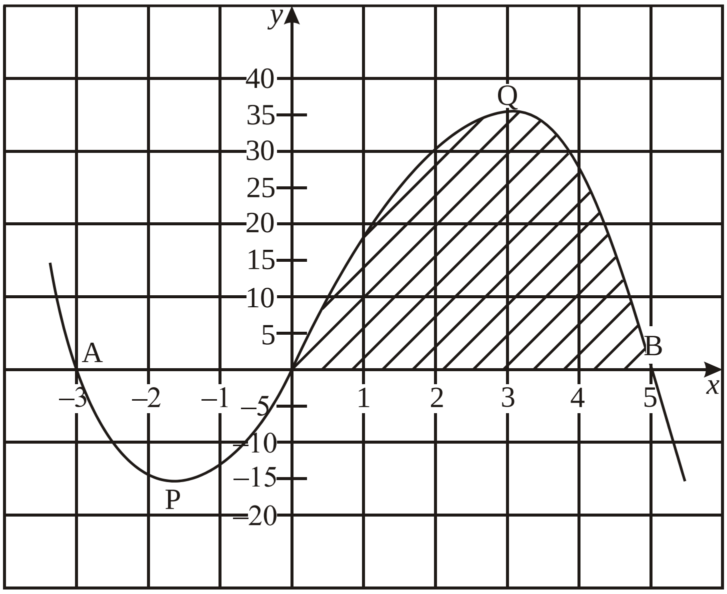
Name:_________________

Answer the following questions :-

1. The diagram below shows part of the graph of the function

$$f : x \mapsto -x^3 + 2x^2 + 15x.$$



The graph intercepts the x-axis at A(–3, 0), B(5, 0) and the origin, O. There is a minimum point at P and a maximum point at Q.

(a) The function may also be written in the form $f : x \mapsto -x(x-a)(x-b)$, where $a < b$. Write down the value of

(i) a; (ii) b.

(b) Find

(i) $f'(x)$;

(ii) the **exact** values of x at which $f'(x) = 0$;

(iii) the value of the function at Q.

(c) (i) Find the equation of the tangent to the graph of f at O.

(ii) This tangent cuts the graph of f at another point. Give the x-coordinate of this point.

(d) Determine the area of the shaded region.

2. *Radian measure is used, where appropriate, throughout the question.*

Consider the function $y = \dfrac{3x-2}{2x-5}$.

The graph of this function has a vertical and a horizontal asymptote.

(a) Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote.

(b) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$, simplifying the answer as much as possible.

(c) How many points of inflexion does the graph of this function have?

3. Let the function f be defined by $f(x) = \dfrac{2}{1+x^3}$, $x \neq -1$.

(a) (i) Write down the equation of the vertical asymptote of the graph of f.

(ii) Write down the equation of the horizontal asymptote of the graph of f.

(iii) Sketch the graph of f in the domain $-3 \le x \le 3$.

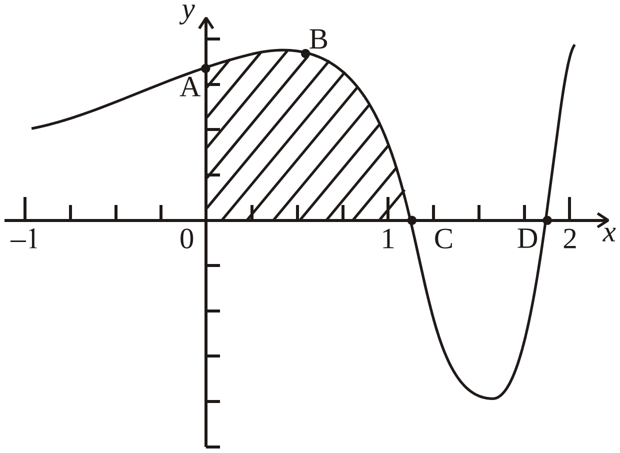
(b) (i) Using the fact that $f'(x) = \dfrac{-6x^2}{(1+x^3)^2}$, show that the second derivative

$$f''(x) = \frac{12x\left(2x^3-1\right)}{(1+x^3)^3}.$$

(ii) Find the x-coordinates of the points of inflexion of the graph of f.

4. The graph of $y = x^3 - 10x^2 + 12x + 23$ has a maximum point between $x = -1$ and $x = 3$. Find the coordinates of this maximum point.

5. The diagram below shows a sketch of the graph of the function $y = \sin(\mathrm{e}^x)$ where $-1 \le x \le 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.



(a) Find the coordinates of A.

(b) The coordinates of C may be written as ($\ln k$, 0). Find the **exact** value of k.

(c) (i) Write down the y-coordinate of B.

(ii) Find $\frac{dy}{dx}$.

(iii) Hence, show that at B, $x = \ln \frac{\pi}{2}$.

(d) (i) Write down the integral which represents the shaded area.

(ii) Evaluate this integral.

(e) (i) Copy the above diagram into your answer booklet. (There is no need to copy the shading.) On your diagram, sketch the graph of $y = x^3$.

(ii) The two graphs intersect at the point P. Find the x-coordinate of P.

6. Consider the function $h(x) = x^{\frac{1}{5}}$.

(i) Find the equation of the tangent to the graph of h at the point where $x = a$, ($a \neq 0$). Write the equation in the form $y = mx + c$.

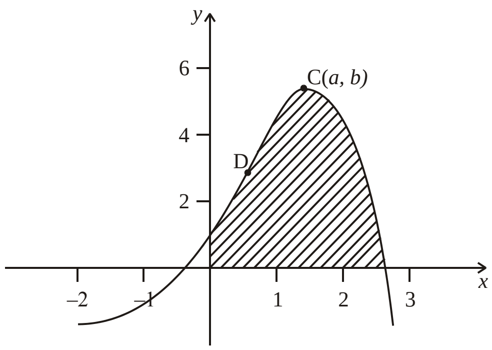
(ii) Show that this tangent intersects the x-axis at the point ($-4a$, 0).

7. Consider the function $f(x) = \cos x + \sin x$.

(a) (i) Show that $f(-\frac{\pi}{4}) = 0$.

(ii) Find in terms of π, the smallest **positive** value of x which satisfies $f(x) = 0$.

The diagram shows the graph of $y = e^x(\cos x + \sin x)$, $-2 \leq x \leq 3$. The graph has a maximum turning point at C(a, b) and a point of inflexion at D.



(b) Find $\frac{dy}{dx}$.

(c) Find the **exact** value of a and of b.

(d) Show that at D, $y = \sqrt{2}e^{\frac{\pi}{4}}$.

(e) Find the area of the shaded region.

8. Consider the function $f(x) = 1 + e^{-2x}$.

(a) (i) Find $f'(x)$.

(ii) Explain briefly how this shows that $f(x)$ is a decreasing function for all values of x (*ie* that $f(x)$ always decreases in value as x increases).

Let P be the point on the graph of f where $x = -\frac{1}{2}$.

(b) Find an expression in terms of e for

(i) the y-coordinate of P;

(ii) the gradient of the tangent to the curve at P.

(c) Find the equation of the tangent to the curve at P, giving your answer in the form $y = ax + b$.

(d) (i) Sketch the curve of f for $-1 \le x \le 2$.

(ii) Draw the tangent at $x = -\frac{1}{2}$.