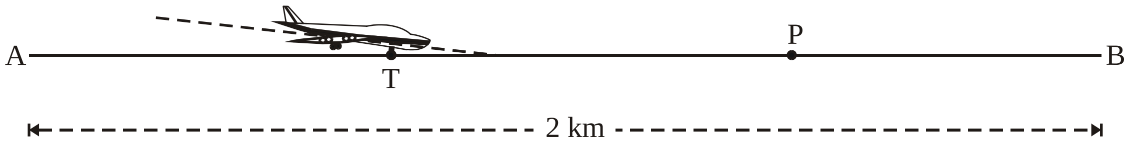
Name:_________________

Answer the following questions :-

1. The main runway at *Concordville* airport is 2 km long. An airplane, landing at *Concordville*, touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



Not to scale

As the airplane slows down, its distance, s, from A, is given by

$$s = c + 100t - 4t^2,$$

where t is the time in seconds after touchdown, and c metres is the distance of T from A.

(a) The airplane touches down 800 m from A, (*ie* $c = 800$).

(i) Find the distance travelled by the airplane in the first 5 seconds after touchdown.

(ii) Write down an expression for the velocity of the airplane at time t seconds after touchdown, and hence find the velocity after 5 seconds.

The airplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker;

(iv) the distance from P to A.

(b) Show that if the airplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway.

2. A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 2 seconds a safety rope slows him down. The height h metres of the rock-climber after t seconds of the fall is given by:

$$h = 50 - 5t^2, \qquad 0 \le t \le 2$$

$$h = 90 - 40t + 5t^2, \qquad 2 \le t \le 5$$

(a) Find the height of the rock-climber when $t = 2$.

(b) Sketch a graph of h against t for $0 \le t \le 5$.

(c) Find $\frac{dh}{dt}$ for:

(i) $0 \le t \le 2$ (ii) $2 \le t \le 5$

(d) Find the velocity of the rock-climber when $t = 2$.

(e) Find the times when the velocity of the rock-climber is zero.

(f) Find the minimum height of the rock-climber for $0 \le t \le 5$.

3. A ball is thrown vertically upwards into the air. The height, h metres, of the ball above the ground after t seconds is given by

$$h = 2 + 20t - 5t^2,\ t \ge 0$$

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(b) Show that the height of the ball after one second is 17 metres.

(c) At a later time the ball is **again** at a height of 17 metres.

(i) Write down an equation that t must satisfy when the ball is at a height of 17 metres.

(ii) Solve the equation **algebraically**.

(d) (i) Find $\frac{\mathrm{d}h}{\mathrm{d}t}$.

(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball.

4. A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t},\ t \ge 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$; (ii) $t = 10$.

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$?

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

(iii) Explain the relationship between the answers to parts (i) and (ii).

(d) Let y metres be the distance fallen after t seconds.

(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.

(ii) Given that $y = 0$ when $t = 0$, find the value of k.

(iii) Find the time required to fall 250 m, giving your answer correct to **four** significant figures.

5. In this question, s represents displacement in metres, and t represents time in seconds.

(a) The velocity v m s^{-1} of a moving body may be written as $v = \frac{ds}{dt} = 30 - at$, where a is a constant. Given that $s = 0$ when $t = 0$, find an expression for s in terms of a and t.

Trains approaching a station start to slow down when they pass a signal which is 200 m from the station.

(b) The velocity of Train 1 t seconds after passing the signal is given by $v = 30 - 5t$.

(i) Write down its velocity as it passes the signal.

(ii) Show that it will stop before reaching the station.

(c) Train 2 slows down so that it stops at the station. Its velocity is given by $v = \frac{ds}{dt} = 30 - at$, where a is a constant.

(i) Find, in terms of a, the time taken to stop.

(ii) Use your solutions to parts (a) and (c)(i) to find the value of a.

6. An aircraft lands on a runway. Its velocity v m s^{-1} at time t seconds after landing is given by the equation $v = 50 + 50e^{-0.5t}$, where $0 \le t \le 4$.

(a) Find the velocity of the aircraft

(i) when it lands;

(ii) when $t = 4$.

(b) Write down an integral which represents the distance travelled in the first four seconds.

(c) Calculate the distance travelled in the first four seconds.

After four seconds, the aircraft slows down (decelerates) **at a constant rate** and comes to rest when $t = 11$.

(d) **Sketch** a graph of velocity against time for $0 \le t \le 11$. Clearly label the axes and mark on the graph the point where $t = 4$.

(e) Find the constant rate at which the aircraft is slowing down (decelerating) between $t = 4$ and $t = 11$.

(f) Calculate the distance travelled by the aircraft between $t = 4$ and $t = 11$.