Name:________________

1. Draw a sketch of the graph of the function $f(x), x \in \mathbb{R}$, where,

(a) $f(1) = 2, f'(1) = 0, f(3) = -2, f'(3) = 0, f'(x) < 0$ for $1 < x < 3$ and $f'(x) > 0$ for $x > 3$ and $x < 1$.

(b) $f'(2) = 0, f(2) = 0$, $f'(x) > 0$ for $0 < x < 2$ and $x > 2$, $f'(x) < 0$ for $x < 0$ and $f(0) = -4$.

(c) $f(4) = f(0) = 0$, $f'(0) = f'(3) = 0$, $f'(x) > 0$ for x > 3 and $f'(x) < 0$ for $x < 0$ and $0 < x < 3$.

2. Find the coordinates and nature of the stationary points for the following:

(e) $f(x) = 3 + 9x - 3x^2 - x^3$ (f) $y = (x-1)(x^2-4)$

(i) $y = (x-1)^2(x+1)$ (j) $y = x\sqrt{x} - x, x \geq 0$

(k) $g(x) = x + \frac{4}{x}, x \neq 0$ (l) $f(x) = x^2 + \frac{1}{x^2}, x \neq 0$

3. Sketch the following functions:

(a) $y = 5 - 3x - x^2$

(c) $f(x) = x^3 + 6x^2 + 9x + 4$ (d) $f(x) = x^3 - 4x$

(e) $f(x) = \frac{1}{3}x^3 - x^2 + 4$ (f) $y = 4x^3 - x^4$

4. Find and describe the nature of all stationary points and points of inflection for the function $f(x) = x^3 + 3x^2 - 9x + 2$.

5. A function f is defined by $f: x \mapsto e^{-x}\sin x$, where $0 \le x \le 2\pi$.

(a) Find i. $f'(x)$ ii. $f''(x)$

(b) Find the values of x for which

i. $f'(x) = 0$ ii. $f''(x) = 0$

(c) Using parts (a), and (b), find the points of inflection and stationary points for f.

(d) Hence, sketch the graph of f.

6. A function f is defined by $f: x \mapsto xe^{-x}$, where $x > 0$.

(a) Find i. $f'(x)$ ii. $f''(x)$

(b). Find the values of x for which

i. $f'(x) = 0$ ii. $f''(x) = 0$

(c) Using parts (a) and (b), find the points of inflection and stationary points for f.

(d) Hence, sketch the graph of f.

7. (a) Find the maximum value of the function $y = 6x - x^2, 4 \le x \le 7$.

(b) Find the minimum value of the function $y = 6x - x^2, 2 \le x \le 6$.

8. The function $f(x) = ax^3 + bx^2 + cx + d$ has turning points at $\left(-1, -\frac{13}{3}\right)$ and $(3, -15)$.

Sketch the graph of the curve $y = f(x)$.

9. The function $f: x \mapsto \mathbb{R}$, where $f(x) = ax^5 + bx^3 + cx$ has stationary points at $(-2, 64)$ $(2, -64)$ and $(0, 0)$. Find the values of a, b and c and hence sketch the graph of f.

10. The curve with equation $y = ax^3 + bx^2 + cx + d$ intersects the x-axis at $x = 1$ and cuts the y-axis at $(0, -34)$. Given that the curve has turning points at $x = 3$ and $x = 5$, determine the values of a, b, c and d. Sketch this curve.